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Collisionless Boltzmann equation for systems obeying Tsallis distribution

E.K. Lenzi[1,a] and R.S. Mendes[2]

[1] Centro Brasileiro de Pesquisas Físicas, R. Dr. Xavier Sigaud 150, 22290-180 Rio de Janeiro-RJ, Brazil
[2] Departamento de Física, Universidade Estadual de Maringá, Av. Colombo 5790, 87020-900 Maringá-PR, Brazil

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Abstract. The collisionless Boltzmann equation is generalized herein using the Green function theory proposed recently by A.K. Rajagopal *et al.* [Phys. Rev. Lett. **80**, 3911 (1998)] to describe nonextensive systems based on Tsallis formalism. Its invariance with the nonextensive index, q, and some conservative laws useful in the description of sound propagation are verified. In this context, the Wigner distribution has also been introduced. Furthermore, the formalism developed in the research is applied for fermionic system, to obtain the dynamic dielectric response function.

PACS. 05.70.Ce Thermodynamic functions and equations of state – 05.30.-d Quantum statistical mechanics – 05.20.-y Classical statistical mechanics – 05.30.Ch Quantum ensemble theory

1 Introduction

The usual statistical mechanics has recently been extended through the employment of the nonextensive Tsallis entropy [1–3]

$$S_q = \frac{1-\mathrm{Tr}\hat{\rho}^q}{q-1}\,, \tag{1}$$

where q and $\hat{\rho}$ are respectively a real parameter and the density matrix. This extension has been applied in many physical situations, Lévy-type anomalous superdiffusion [4], Euler turbulence [5], gravitating systems [5,6], anomalous relaxation through electron-phonon interaction [7], ferrofluid-like systems [8], nonlinear dissipative dynamical systems [9], among others. It may not be out of place to mention here some important formal developments such as linear response theory [10], Green functions [11,12], variational and perturbative methods [13,14], and generalization of Laplace transform [15] used in connection with nonextensive phenomena and Tsallis statistics. In the above context, it is natural to study how the usual Boltzmann equation, which plays an important role in the analysis of transport phenomena [16,17], can be incorporated in the Tsallis framework. In fact, the usual Boltzmann equation accomplishing the Tsallis statistics may develop new possibilities in the exploration of dynamical aspects of systems with long-range interaction [1–3] and fractal boundary conditions in metals [18,19].

The purpose of this work is to develop the collisionless-like Boltzmann equation by incorporating the general Green functions theory [11,12] based on Tsallis statistics. More precisely, here we generalize the technique and approach adopted by Kadanoff-Baym [16] for collisionless Boltzmann equation, so the present derivation is an extension of their results by incorporating the Tsallis statistics. Present development may have implications in the description of possible future experiments, evolving fermionic and bosonic systems worked in references [11,12], and in high frequency vibrations in a collisionless electronic plasma [20,21]. In addition, this work can be used to give a basis for the analysis performed in the recent works [7,22] based on the nonextensive framework. Furthermore, we apply the formalism developed in this research to fermionic systems in order to obtain some quantities of physical interest such as change of density and dielectric function. We also find the expressions for the average number and for the internal energy in the limit of high temperature and low temperature employing the normalized version of the Tsallis statistics for q less than one.

[a] e-mail: eklenzi@cbpf.br

2 Boltzmann transport-like equation

As it is well known, there is a class of disturbances not conveniently described by the usual equilibrium Green functions [16]: the disturbance produced by an externally applied force field, $\mathbf{F}(\mathbf{r},t) = -\boldsymbol{\nabla} U(\mathbf{r},t)$. Many interesting physical phenomena appear as response of systems to external disturbances of this kind. For instance, in ordinary gas, a slowly varying $U(\mathbf{r},t)$ produces sound waves. Similar features are also expected to appear in nonextensive systems. Concepts based on Tsallis statistics should be thus employed to investigate the above issues.

The generalized canonical density matrix $\hat{\rho}$, obtained from the maximum entropy principle, taking into account Tsallis entropy and the q-expectations averages $\langle \hat{H} \rangle_q = \text{Tr}\hat{\rho}^q \hat{H}/\text{Tr}\hat{\rho}^q$ [3] and $\langle \hat{N} \rangle_q = \text{Tr}\hat{\rho}^q \hat{N}/\text{Tr}\hat{\rho}^q$ [3] , is given by

$$\hat{\rho} = \exp_q \left[\frac{-\beta}{\text{Tr}\hat{\rho}^q} \left(\hat{H} - \mu \hat{N} - \langle \hat{H} - \mu \hat{N} \rangle_q \right) \right] \Big/ Z_q$$

$$Z_q = \text{Tr} \left(\exp_q \left[\frac{-\beta}{\text{Tr}\hat{\rho}^q} \left(\hat{H} - \mu \hat{N} - \langle \hat{H} - \mu \hat{N} \rangle_q \right) \right] \right), \quad (2)$$

where $\exp_q [x] = [1 + (1 - q)x]^{1/(1-q)}$ is the generalized Tsallis exponential, and Z_q is the partition function. Now, accomplishing the q-expectation average of the operator $\hat{X}(\mathbf{R}, t)$ by using the interaction picture, we have that

$$\langle \hat{X}(\mathbf{R}, t) \rangle_{U,q} = \frac{\sum_i \hat{\rho}_i^q \langle i, t_0 | \hat{X}_U(\mathbf{R}, t) | i, t_0 \rangle}{\sum_i \hat{\rho}_i^q}$$

$$= \langle X_U(\mathbf{R}, T) \rangle_q \ , \quad (3)$$

$\hat{X}_U(\mathbf{R}, t) = \mathcal{V}(t)^{-1} \hat{X}(\mathbf{R}, t) \mathcal{V}(t)$ with $\mathcal{V}(t) = \text{T}\Big(\exp \Big[-\text{i} \int_{t_0}^t \text{d}t' \int \text{d}^3 r' \hat{n}(r', t') U(r', t') \Big] \Big)$ (T is the time ordering operator), following the approach employed in [16]. Note that $\mathcal{V}(t)$ is chosen in order to incorporate the external disturbance $\hat{H}'(t) = \int \text{d}^3 r \ \ \hat{n}(r, t) U(r, t)$, where $\hat{n}(r, t) = \Psi_U^\dagger(r, t) \Psi_U(r, t)$. Thus, the Green functions developed in [12], within the normalized averages [3], may be rewritten in a suitable form,

$$\tilde{G}^{(q)}(1, 1'; U) = \frac{1}{\text{i}} \langle \text{T}(\Psi_U^\dagger(1') \Psi_U(1)) \rangle_q \quad (4)$$

and

$$\tilde{G}^{(q)}(12, 1'2'; U) = \left(\frac{1}{\text{i}} \right)^2 \langle \text{T}(\Psi_U(1) \Psi_U(2) \Psi_U^\dagger(2) \Psi_U^\dagger(1)) \rangle_q \ . \quad (5)$$

In terms of the above Green functions, we may describe the response of a system, initially in thermodynamical equilibrium, to an applied disturbance $U(\mathbf{r}, t)$.

Employing the above development, the average density at point $(\mathbf{R}, T)$ is given by

$$\langle \hat{n}(\mathbf{R}, T) \rangle_{U,q} = \langle \Psi_U^\dagger(\mathbf{R}, T) \Psi_U(\mathbf{R}, T) \rangle_q$$

$$= \pm \text{i} \tilde{G}_<^{(q)}(\mathbf{R}, T, \mathbf{R}, T; U) \ , \quad (6)$$

and the density current at the same point is

$$\langle \hat{J}(\mathbf{R}, T) \rangle_{U,q} = \left\{ \frac{\nabla - \nabla'}{2m\text{i}} \left[\pm \text{i} \tilde{G}_<^{(q)}(\mathbf{R}, T, \mathbf{R}', T; U) \right] \right\}_{\mathbf{R} = \mathbf{R}'} . \quad (7)$$

In these equations and in the following discussion, the upper sign (+) if for the bosonic case and the lower one is for fermions. The conservation laws for the number of particles, the energy and the total momentum are preserved here as well as in the usual case. In terms of $\langle \hat{n}(\mathbf{R}, T) \rangle_{U,q}$ and $\langle \hat{J}(\mathbf{R}, T) \rangle_{U,q}$, main results for the derivation of sound propagation are

$$\frac{\partial}{\partial T} \langle \hat{n}(\mathbf{R}, T) \rangle_{U,q} + \nabla \cdot \langle \hat{J}(\mathbf{R}, T) \rangle_{U,q} = 0 \ , \quad (8)$$

$$\frac{\text{d}}{\text{d}T} \langle \hat{H}(T) \rangle_{U,q} + \int \text{d}R \, [\nabla U(\mathbf{R}, T)] \cdot \langle \hat{J}(\mathbf{R}, T) \rangle_{U,q} = 0 \ , \quad (9)$$

$$\frac{\text{d}}{\text{d}t} \langle \hat{\mathbf{P}}(t) \rangle_{U,q} + \int \text{d}R \, [\nabla U(\mathbf{R}, t)] \cdot \langle \hat{n}(\mathbf{R}, t) \rangle_{U,q} = 0 \ . \quad (10)$$

The above equations can be verified directly by using the Heisenberg equation for $\Psi^\dagger$ and Ψ as well as equations (6) and (7).

In the context of Tsallis statistics, the Wigner distribution $f_q(\mathbf{p}, \mathbf{R}, T)$ (with $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_{1'}$, $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_{1'})/2$, $t = t_1 - t_{1'}$ and $T = (t_1 + t_{1'})/2$) may be defined as

$$f_q(\mathbf{p}, \mathbf{R}, T) = \int \frac{\text{d}\omega}{2\pi} \tilde{G}_<^{(q)}(\mathbf{p}, \omega, \mathbf{R}, T; U)$$

$$= \int \text{d}^3 \ r \text{e}^{-\text{i}\mathbf{p} \cdot \mathbf{r}} \left\langle \Psi_U^\dagger \left(\mathbf{R} - \frac{\mathbf{r}}{2}, T \right) \times \Psi_U \left(\mathbf{R} + \frac{\mathbf{r}}{2}, T \right) \right\rangle_q \ . \quad (11)$$

Similar to the standard case, $f_q(\mathbf{p}, \mathbf{R}, T)$ leads to the generalized q-particle density

$$\int \frac{\text{d}^3 p}{(2\pi)^3} f_q(\mathbf{p}, \mathbf{R}, T) = \langle \Psi_U^\dagger(\mathbf{R}, T) \, \Psi_U(\mathbf{R}, T) \rangle_q$$

$$= \langle \hat{n}(\mathbf{R}, T) \rangle_q \ , \quad (12)$$

and the generalized q-particle current

$$\langle \hat{J}(\mathbf{R}, T) \rangle_{U,q} = \int \frac{\text{d}^3 p}{(2\pi)^3} \frac{\mathbf{p}}{m} f_q(\mathbf{p}, \mathbf{R}, T) \ . \quad (13)$$

The above definition of the distribution function $f_q(\mathbf{p}, \mathbf{R}, T)$ will enable us to see the relationship between Green functions, transport equations and the generalized collisionless Boltzmann equation.

The generalized collisionless Boltzmann equation may be obtained by using the equation of motion

$$\left(\text{i} \frac{\partial}{\partial t_1} + \frac{\nabla_1^2}{2m} - U(1) \right) \tilde{G}^{(q)}(1, 1'; U) =$$

$$\delta(1 - 1') \pm \int \text{d}\mathbf{2} V(\mathbf{1} - \mathbf{2}) \tilde{G}^{(q)}(12, 1'2^+; U) \quad (14)$$

($\mathrm{d}\mathbf{2} = \mathrm{d}^3 r_2 \mathrm{d}t_2$, $V(\mathbf{1}-\mathbf{2}) = v(\mathbf{r}_1 - \mathbf{r}_2)\delta(t_1 - t_2)$) and the Hartree approximation $\tilde{G}^{(q)}(12, 1'2'; U) = \tilde{G}^{(q)}(1, 1'; U)\tilde{G}^{(q)}(2, 2'; U)$, physically motivated by the propagation interpretation of $\tilde{G}^{(q)}(1, 1'; U)$ and $\tilde{G}^{(q)}(12, 1'2'; U)$ as in [11,12,16]. Thus, we obtain from equation (14) and the Hartree approximation, after some simplifications, the equation

$$\left(\mathrm{i}\frac{\partial}{\partial t_1} + \frac{\nabla_1^2}{2m} - U_{\mathrm{eff}}(1)\right)\tilde{G}^{(q)}(1, 1'; U) = \delta(1 - 1') \ , \quad (15)$$

where $U_{\mathrm{eff}}(\mathbf{R}, T) = U(\mathbf{R}, T) \pm \mathrm{i}\int \mathrm{d}\mathbf{R}' v(\mathbf{R} - \mathbf{R}') \times \tilde{G}^{(q)}_{<}(\mathbf{R}', T; \mathbf{R}', T)$. By taking the difference of equation (15) in the variables 1 and $1'$, we find

$$\left[\mathrm{i}\left(\frac{\partial}{\partial t_1} + \frac{\partial}{\partial t_{1'}}\right) + (\nabla_1 + \nabla_{1'}) \cdot \left(\frac{\nabla_1 - \nabla_{1'}}{2m}\right) - [U_{\mathrm{eff}}(1) - U_{\mathrm{eff}}(1')]\right]\tilde{G}^{(q)}(1, 1'; U) = 0. \quad (16)$$

Considering $t_{1'} = t_1^+ = T$ and expressing equation (16) in terms of $\mathbf{r} = \boldsymbol{r}_1 - \boldsymbol{r}_{1'}$, $\mathbf{R} = (\boldsymbol{r}_1 + \boldsymbol{r}_{1'})/2$, and using equation (11), we verify that

$$\left(\frac{\partial}{\partial T} + \frac{\nabla_{\mathbf{R}} \cdot \nabla_{\mathbf{r}}}{im} - \frac{1}{\mathrm{i}}\left[U_{\mathrm{eff}}(\mathbf{R} + \frac{\mathbf{r}}{2}, T) - U_{\mathrm{eff}}(\mathbf{R} - \frac{\mathbf{r}}{2}, T)\right]\right)\int \frac{\mathrm{d}^3 p'}{(2\pi)^3}\mathrm{e}^{\mathrm{i}\mathbf{p}'\cdot\mathbf{r}} f_q(\mathbf{p}', \mathbf{R}, T) = 0. \quad (17)$$

Now, supposing that $U_{\mathrm{eff}}(\mathbf{R}, T)$ varies slowly in $\mathbf{R}$, the generalized collisionless Boltzmann equation is given by

$$\left[\frac{\partial}{\partial T} + \frac{\mathbf{p}}{2m} \cdot \nabla_{\mathbf{R}} - \nabla_{\mathbf{R}} U_{\mathrm{eff}}(\mathbf{R}, T) \cdot \nabla_{\mathbf{p}}\right] \times f_q(\mathbf{p}, \mathbf{R}, T) = 0, \quad (18)$$

with $U_{\mathrm{eff}}(\mathbf{R}, T) = U(\mathbf{R}, T) + \int \mathrm{d}\mathbf{R}' v(\mathbf{R} - \mathbf{R}') \times \int \frac{\mathrm{d}^3 p'}{(2\pi)^3} f_q(\mathbf{p}', \mathbf{R}', T)$. Equation (18) preserves its usual form for an arbitrary q as the Bogoliubov inequality [13]. Although the generalized collisionless Boltzmann equation has the same formal aspect as the usual one [16], it leads to different results. In fact, the systems of interest in both cases are promoted by different features, *i.e.*, the boundary condition for $f_q(\mathbf{R}, \mathbf{R}', T)$ with $q \neq 1$ and $q = 1$ are different. In order to make the differences between $f_q(\mathbf{R}, \mathbf{R}', T)$ and the usual one evident, we work equation (18) in the following section by considering q near unity and the linearization of the Hartree approximation.

3 Application

Simple textbook examples, such as free particle, the harmonic oscillator, the non-interacting scalar field, etc., do not exhibit any kind of nonextensive behavior. It is not necessary to introduce a generalized statistical mechanics to deal with these systems. However, these simple examples are very useful in order to illustrate how the formalism works. And they are even more instructive when they provide exactly solvable cases. Thus, we start our application by considering the small potential $U(\mathbf{R}, T)$ and q less than one, *i.e.*, we solve equation (18) in the random phase approximation with q less than one.

To solve equation (18) we must know the initial condition. Let us suppose that, in the limit $T \to -\infty$, the disturbances vanish; then, $f_q(\mathbf{p}, \mathbf{R}, T)$ is given by the equilibrium condition. In addition, employing the Hartree approximation we have that

$$\lim_{T\to-\infty} f_q(\mathbf{p}, \mathbf{R}, T) = f_q(E(p)), \quad (19)$$

with $f_q(E(p))$ defined in [12] as:

$$f_q(E(p)) = \int_C \mathrm{d}u \tilde{K}_q^{(2)}(u)\frac{\tilde{Z}_1(-u(1-q)\tilde{\beta}, \mu)}{\mathrm{e}^{-u(1-q)\tilde{\beta}(E(p)-\mu)} + 1} , \quad (20)$$

where $\tilde{K}_q^{(2)}(u)$ is given by

$$K_q^{(2)}(u) = \frac{\mathrm{i}}{2\pi Z_q}\Gamma\left(\frac{2-q}{1-q}\right)\exp(-u)(-u)^{-1/(1-q)} , \quad (21)$$

with $\tilde{\beta} = \beta/\mathrm{Tr}\rho^q$, $E(p) = p^2/(2m) + \langle\hat{n}\rangle_q \int \mathrm{d}r v(r)$ and $\tilde{Z}_1$ being the corresponding conventional partition function multiplied by $\mathrm{e}^{-u\tilde{\beta}(1-q)(\tilde{U}_q - \mu\tilde{N}_q)}$.

The first order in $U(\mathbf{R}, T)$ leads to

$$f_q(\mathbf{p}, \mathbf{R}, T) = f_q(E(p)) + \delta f_q(\mathbf{p}, \mathbf{R}, T) \ , \quad (22)$$

where $\delta f_q(\mathbf{p}, \mathbf{R}, T) = \int_{-\infty}^{T}\int \mathrm{d}T'\mathrm{d}R' \frac{\delta}{\delta U} f_q(\mathbf{p}, \mathbf{R} - \mathbf{R}', T - T')U(\mathbf{R}', T')$. This equation defines the linear response in the real time domain. Due to the smallness of $U(\mathbf{R}, T)$, substituting equation (22) into equation (18) and taking into consideration the case in which $U(\mathbf{R}, T)$ has the form of $U(\mathbf{R}, T) = U(k, \omega)\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{R} - \mathrm{i}\omega T}$, we calculate two physically interesting quantities. In the random phase approximation, the first quantity is the change of density,

$$\delta\langle\hat{n}(\mathbf{k}, \omega)\rangle_q = \int \frac{\mathrm{d}^3 p}{(2\pi)^3}\delta f_q(\mathbf{p}, \mathbf{k}, \omega) = \frac{U(k, \omega)}{1 - v(k)\frac{\delta}{\delta U}\langle\hat{n}(\mathbf{k}, \omega)\rangle_q}\frac{\delta}{\delta U}\langle\hat{n}(\mathbf{k}, \omega)\rangle_q, \quad (23)$$

with

$$\frac{\delta}{\delta U}\langle\hat{n}(\mathbf{k}, \omega)\rangle_q = \int \frac{\mathrm{d}^3 p}{(2\pi)^3}\frac{f_q(E(\mathbf{p} - \mathbf{k}/2)) - f_q(E(\mathbf{p} + \mathbf{k}/2))}{\omega - \mathbf{k}\cdot\mathbf{p}/m} . \quad (24)$$

Another function of direct interest is the dynamic dielectric response function, K_q, which is given by

$$K_q(k, \omega) = \left[1 - v(k)\frac{\delta}{\delta U}\langle\hat{n}(\mathbf{k}, \omega)\rangle_q\right]^{-1} . \quad (25)$$

Now, let us analyze systems of charged particles employing the random phase approximation developed above. Interaction is through the Coulomb potential, $v(R) = \mathrm{e}^2/R$. After some calculations and considering the limit in which the disturbance varies so slowly in space that $(\mathbf{k}\cdot\mathbf{p})^2 \ll \omega^2$, we obtain

$$\frac{\delta}{\delta U}\langle \hat{n}(\mathbf{k},\omega)\rangle_q = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \times \left[f_q\left(\frac{(\mathbf{p}-\mathbf{k}/2)^2}{2m}\right) - f_q\left(\frac{(\mathbf{p}+\mathbf{k}/2)^2}{2m}\right)\right] \times \left[\frac{\mathbf{k}\cdot\mathbf{p}}{m\omega^2} + \frac{1}{\omega^5}\left(\frac{\mathbf{k}\cdot\mathbf{p}}{m}\right)^3\right]. \quad (26)$$

Shifting the origin of p integration in equation (26) and keeping only the terms up to order k^4, we have

$$\frac{\delta}{\delta U}\langle \hat{n}(\mathbf{k},\omega)\rangle_q = \frac{\langle \hat{n}\rangle_q}{m\omega^2}\left[1 + \frac{k^2}{\omega^2}\langle \mathbf{v^2}\rangle_q\right], \quad (27)$$

where

$$\langle \hat{n}\rangle_q = \int \frac{\mathrm{d}^3 p}{(2\pi)^3}\int_C \mathrm{d}u \tilde{K}_q^{(2)}(u) \frac{\tilde{Z}_1(-u(1-q)\tilde{\beta},\mu)}{\mathrm{e}^{-u(1-q)\tilde{\beta}(E(p)-\mu)}+1} \quad (28)$$

and

$$\langle \mathbf{v^2}\rangle_q = \frac{1}{\langle \hat{n}\rangle_q}\int \frac{\mathrm{d}^3 p}{(2\pi)^3} \times \int_C \mathrm{d}u \frac{p^2}{m^2}\tilde{K}_q^{(2)}(u) \frac{\tilde{Z}_1(-u(1-q)\tilde{\beta},\mu)}{\mathrm{e}^{-u(1-q)\tilde{\beta}(E(p)-\mu)}+1}. \quad (29)$$

For $\mathbf{k}\cdot\mathbf{p} \ll \omega^2$, the dielectric function (25) can be found by substituting equation (27), so we obtain

$$K_q(k,\omega) = \omega^2\left[\omega^2 - \frac{4\pi \mathrm{e}^2}{k^2}\langle \hat{n}\rangle_q\left(1 + \frac{k^2}{\omega^2}\langle \mathbf{v^2}\rangle_q\right)\right]^{-1}, \quad (30)$$

with $\langle \hat{n}\rangle_q$ and $\langle \mathbf{v^2}\rangle_q$ defined in equations (28) and (29). We note that there are poles in the approximated response function (30) at $\omega^2 = 4\pi\mathrm{e}^2\langle \hat{n}\rangle_q/m + k^2\langle \mathbf{v^2}\rangle_q$. These poles, which depend on q, can indicate possible excitation, or resonant response from the system. In Figure 1, we show the behavior obtained from ω^2 for typical q values in order to illustrate the results obtained within the generalized case ($q \neq 1$). Resonance response also occurs due to the change of density $\delta\langle \hat{n}(\mathbf{k},\omega)\rangle_q$. Therefore, it corresponds to a possible density oscillation of the system. This resonance is a generalization of plasma oscillation in the context of Tsallis statistics.

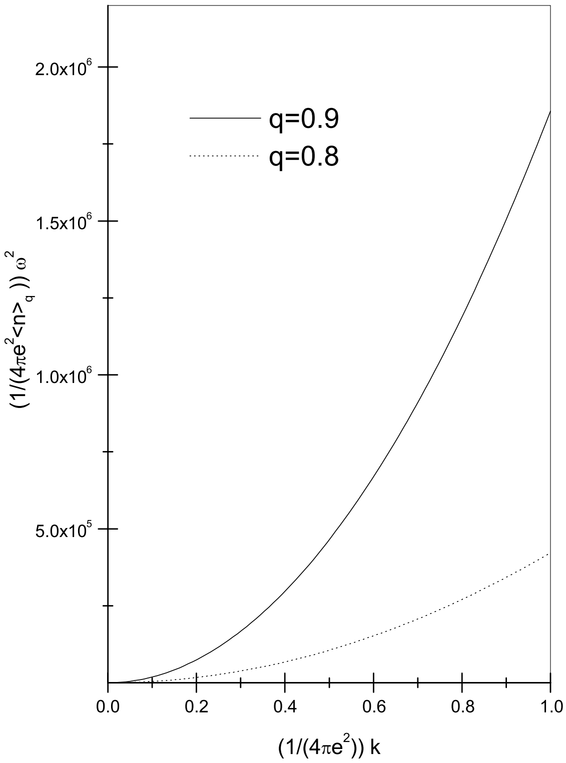

Fig. 1. In this figure we plot $(1/(4\pi\mathrm{e}^2\langle \hat{n}\rangle_q))\omega^2$ *versus* $(1/(4\pi\mathrm{e}^2))k$ by considering for simplicity $1/\tilde{\beta} = 1272.5$, $m = 1$, $\mathrm{V} = 1$ and $\langle \hat{n}\rangle_q = 0.0004$.

In the classical limit for $q < 1$, the expressions (28) and (29) turns out to be

$$\langle \mathbf{v^2}\rangle_q = \frac{1}{Z_q}\frac{3}{m\langle \hat{n}\rangle_q\tilde{\beta}}\left(\frac{m}{2\pi(1-q)\tilde{\beta}}\right)^{3/2} \times \Gamma\left(\frac{2-q}{1-q}\right)\sum_{n=0}^{\infty}\frac{1}{n!}\left[V\left(\frac{m}{2\pi(1-q)\tilde{\beta}}\right)^{3/2}\right]^n \times \frac{\left[1+(1-q)\tilde{\beta}\left(\langle \hat{H}-\mu\hat{N}\rangle_q+\mu(n+1)\right)\right]^{1/(1-q)+3/2(n+1)}}{\Gamma\left(\frac{2-q}{1-q}+\frac{3}{2}(n+1)\right)}, \quad (31)$$

and

$$\langle \hat{n}\rangle_q = \frac{\langle \hat{N}\rangle_q}{V} = \frac{1}{Z_q}\left(\frac{m}{2\pi(1-q)\tilde{\beta}}\right)^{3/2} \times \Gamma\left(\frac{1}{1-q}\right)\sum_{n=0}^{\infty}\frac{1}{n!}\left[V\left(\frac{m}{2\pi(1-q)\tilde{\beta}}\right)^{3/2}\right]^n \times \frac{\left[1+(1-q)\tilde{\beta}\left(\langle \hat{H}-\mu\hat{N}\rangle_q+\mu(n+1)\right)\right]^{q/(1-q)+3/2(n+1)}}{\Gamma\left(\frac{1}{1-q}+\frac{3}{2}(n+1)\right)}, \quad (32)$$

with

$$Z_q = \Gamma\left(\frac{2-q}{1-q}\right)\sum_{n=0}^{\infty}\frac{1}{n!}\left[V\left(\frac{m}{2\pi(1-q)\tilde{\beta}}\right)^{3/2}\right]^n \times \frac{\left[1+(1-q)\tilde{\beta}\left(\langle\hat{H}-\mu\hat{N}\rangle_q+\mu n\right)\right]^{1/(1-q)+3/2n}}{\Gamma\left(\frac{1}{1-q}+\frac{3}{2}n\right)}, \quad (33)$$

where $\langle \mathbf{v}^2\rangle_q = (2/m)(\langle\hat{H}\rangle_q/\langle N\rangle_q)$. In addition, in the limit of low temperature we may approximate the expressions for $\langle \mathbf{v^2}\rangle_q$ and $\langle\hat{n}\rangle_q$ by

$$\langle \mathbf{v^2}\rangle_q \approx \frac{1}{\langle\hat{n}\rangle_q}\left[\frac{3}{5m}\langle\hat{n}\rangle_1\epsilon_{\mathrm{F}} + \frac{2}{m}(\mu-\epsilon_{\mathrm{F}})\epsilon_{\mathrm{F}}g(\epsilon_{\mathrm{F}}) + \frac{\pi^2}{2m}\frac{g(\mu)}{(1-q)\tilde{\beta}^2}\left[\frac{\pi^2\langle\hat{n}\rangle_1}{(1-q)\tilde{\beta}\epsilon_{\mathrm{F}}}\left(\frac{\mu}{\epsilon_{\mathrm{F}}}\right)^{1/2}\right]^{-1/2} \times \frac{I_{\frac{2-q}{1-q}}\left(\left[\frac{\pi^2\langle\hat{n}\rangle_1}{(1-q)\tilde{\beta}\epsilon_{\mathrm{F}}}\left(\frac{\mu}{\epsilon_{\mathrm{F}}}\right)^{1/2}\right]^{1/2}\right)}{I_{\frac{1}{1-q}}\left(\left[\frac{\pi^2\langle\hat{n}\rangle_1}{(1-q)\tilde{\beta}\epsilon_{\mathrm{F}}}\left(\frac{\mu}{\epsilon_{\mathrm{F}}}\right)^{1/2}\right]^{1/2}\right)}\right] \quad (34)$$

and

$$\langle\hat{n}\rangle_q \approx \langle\hat{n}\rangle_1 + (\mu-\epsilon_{\mathrm{F}})g(\epsilon_{\mathrm{F}}) + \frac{\pi^2}{6}\frac{1}{(1-q)\tilde{\beta}^2}g'(\mu)\left[\frac{\pi^2\langle\hat{n}\rangle_1}{(1-q)\tilde{\beta}\epsilon_{\mathrm{F}}}\left(\frac{\mu}{\epsilon_{\mathrm{F}}}\right)^{1/2}\right]^{-1/2} \times \frac{I_{\frac{2-q}{1-q}}\left(\left[\frac{\pi^2\langle\hat{n}\rangle_1}{(1-q)\tilde{\beta}\epsilon_{\mathrm{F}}}\left(\frac{\mu}{\epsilon_{\mathrm{F}}}\right)^{1/2}\right]^{1/2}\right)}{I_{\frac{1}{1-q}}\left(\left[\frac{\pi^2\langle\hat{n}\rangle_1}{(1-q)\tilde{\beta}\epsilon_{\mathrm{F}}}\left(\frac{\mu}{\epsilon_{\mathrm{F}}}\right)^{1/2}\right]^{1/2}\right)}, \quad (35)$$

where ϵ_{F} is the Fermi energy, $\langle\hat{n}\rangle_1 = 2/3g(\epsilon_{\mathrm{F}})\epsilon_{\mathrm{F}}$, $g(\epsilon) = 1/(4\pi^2)(2m)^{3/2}\epsilon^{1/2}$ is the density of states, and $\mathrm{I}_\alpha(x)$ is the modified Bessel function of first kind.

Before concluding this section, we call attention for the equation (23), which, in the usual many-body theory ($q = 1$), can be obtained by using the linear response theory. In fact, for $q = 1$ the approach developed here and the one based on the linear response theory are found to be equivalent [16]. In contrast, a possible proof of the equivalence between these approaches (the linear response theory for $q \neq 1$ is developed in [10]) seems to be lost because the boundary condition based on the cyclic property of the trace can not be established for $q \neq 1$, for example $\langle\Psi(r,t)\Psi^\dagger(\overline{r},\overline{t})\rangle_q \neq \langle\Psi^\dagger(\overline{r},\overline{t}-\mathrm{i}\beta)\Psi(r,t)\rangle_q$ for $q \neq 1$. We emphasize that this equivalence is also lost when we employ the random phase approximation. Thus, obtaining equation (23) becomes a hard task (if not impossible) when we employ the linear response theory [10].

4 Summary and conclusions

We have developed the collisionless Boltzmann transport-like equation for nonextensive systems based on Tsallis statistics employing the Green function techniques [11,12] for arbitrary q, and by using the Hartree approximation. We have also applied the present formalism to a fermionic system in order to obtain the dielectric response function and the plasma oscillation accomplishing Tsallis statistics. In addition, we have found the expressions for the average number and for the internal energy in the limit of high temperature and low temperature employing the normalized version of the Tsallis statistics for q less than one. In fact, from a formal point of view, noninteracting and short-range interacting systems are mathematically well posed problems only for $q < 1$ when the degree of freedom are very large. These physical quantities, the dielectric response function and the plasma oscillation, can be used to verify a possible relation between the Tsallis statistics and the anomalous systems (for example, systems with fractal structure [18]). In addition, the present development can also be used to give a firm basis for the analysis performed in [7,22]. We may add that other forms of density matrix (probability distribution), such as presented in [23–25], in the above formalism by considering an appropriate definition of the β parameter. Furthermore, similar to the usual collisionless Boltzmann transport equation, which has been traditionally employed in many physical contexts, we expect that the present work will be of help in the proper analysis of anomalous systems related to nonextensive phenomena.

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